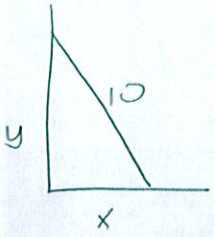
You want to lean a 10 ft long ladder against a wall. For stability, neither end of the ladder can be closer than 4 ft from where the wall meets the ground. What is the smallest possible area between the ladder, the wall and the ground ? **SCORE: ____ / 16 PTS**

HINT: The work required to solve this question can be greatly reduced if you choose the function to be optimized wisely.



MAXIMIZE $A = \frac{1}{2}xy$ (1)

$y = \sqrt{100 - x^2}$

$x^2 + 4^2 = 10^2$

$x = \sqrt{84} = 2\sqrt{21}$

(2) $A = \frac{1}{2}x\sqrt{100-x^2}$

(2) $x \in [4, 2\sqrt{21}]$

$A' = \frac{1}{2}\sqrt{100-x^2} + \frac{1}{2}x\left(\frac{1}{2\sqrt{100-x^2}}\right)(-2x)$ (3)

$= \frac{1}{2\sqrt{100-x^2}}(100 - x^2 - x^2)$

$= \frac{100 - 2x^2}{2\sqrt{100-x^2}}$ (2) EXISTS IN DOMAIN

$= 0$ @ $x = 5\sqrt{2}$ (1)

x	A
4	$4\sqrt{21}$ (1)
$5\sqrt{2}$	25 (1)
$2\sqrt{21}$	$4\sqrt{21}$ (1)

SMALLEST AREA = $4\sqrt{21}$ FT^2 (1) (1)

You use Newton's Method to find the solution of the equation $x = 10\sin x$ with initial approximation $x_0 = 5$. **SCORE: ____ / 6 PTS**

[a] What is the expression for the second approximation x_1 in terms of x_0 ? $x - 10\sin x = 0$

Do NOT use f or f' notation.

$$X_1 = X_0 - \frac{X_0 - 10\sin X_0}{1 - 10\cos X_0}$$ (2)

[b] Write down the decimal values of the second, third and fourth approximations (ie. x_1, x_2, x_3) that Newton's Method generates.

Do NOT round off your answers.

$x_1 =$ 12.94352017 (1) $x_2 =$ 14.05965841 (1) $x_3 =$ -4.061088149 (1)

[c] What is the solution that Newton's Method generates ?

$x =$ -2.852341894 (1)

You use Newton's Method to approximate $\sqrt[5]{100}$ with initial approximation $x_0 = 2$. **SCORE: ____ / 4 PTS**

[a] What is the expression for the second approximation x_1 in terms of x_0 ? $x^5 - 100 = 0$

Do NOT use f or f' notation.

$$X_1 = X_0 - \frac{X_0^5 - 100}{5X_0^4}$$ (2)

[b] Write down the decimal values of the second and third approximations (ie. x_1, x_2) that Newton's Method generates.

Do NOT round off your answers.

$x_1 =$ 2.85 (1) $x_2 =$ 2.583145102 (1)

You wish to use Newton's Method on the graph of f below to find the solution of the equation $f(x) = 0$ with initial approximation x_0 shown. **Graphically** find the approximations x_1 and x_2 **(label them clearly)**. **SCORE: ____ / 4 PTS**

(1) + (1) TANGENTS

(1) + (1) X_1, X_2